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AdS/CFT correspondence and QCD with quarks in fundamental representations

Paul H. Frampton

Department of Physics and Astronomy, University of North Carolina, Chapel Hill, NC 27599, USA

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Abstract

The most straightforward use of AdS/CFT correspondence gives versions of QCD where quarks are in adjoint representations. Using an asymmetric orbifold approach we obtain nonsupersymmetric QCD with four quark flavors in fundamental representations of color.

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The interplay between gauge field theories and string theory is one of the most fertile in high-energy theory. Indeed string theory had its beginnings as an attempted theory of strong interactions [1,2] to be replaced by the gauge field theory of quantum chromodynamics (QCD) [3,4]. Nevertheless, the interconnection between these theories constantly portrays them not as competitors but most recently as dual descriptions. The notion that strings describe some large N_{colors} limit of QCD was suggested already by 't Hooft [5] in 1974. An excellent review is provided by the 1987 book by Polyakov [6]. A large step forward was taken with the identification by Maldacena [7] of the AdS/CFT correspondence. For example, compactifying a superstring on an $AdS_5 \times S^5$ manifold there is a duality of descriptions of either a type IIB superstring in ten spacetime dimensions or an

$\mathcal{N} = 4$ supersymmetric $SU(N)$ gauge field theory in four spacetime dimensions.

This correspondence provides a powerful tool to investigate gauge field theory. Independently of whether the superstring can provide a correct theory of quantum gravity the AdS/CFT correspondence can suggest interesting models for non-gravitational physics. Beyond being merely a tool, it can suggest directions to extend the standard model and even additional particles that may be lying in the TeV regime awaiting discovery [8]. As a tool, it may be used to study QCD and very interesting results have been obtained by Polchinski and Strassler [9–11] about the relationship of QCD to string theory from the AdS/CFT correspondence. This is of special interest in view of the above history because the string theory was originally displaced by QCD thirty years ago.

This recent work on string QCD sheds light on how QCD describes hard scattering by pointlike constituents [12] whereas the string gave only infinitely soft scattering. The resolution comes from the impor-

E-mail address: frampton@physics.unc.edu (P.H. Frampton).

tance of the curved geometry of the fifth dimension in the AdS_5 manifold.

Here we make an observation about the group theory of orbifolding $AdS_5 \times S^5$ and the avoidance of quarks in adjoint representations. In real QCD with gauge group $SU(3)$ the quarks come in six flavors (u, d, s, c, b, t) which transform as fundamental triplets, not adjoint octets, of the color $SU(3)$ gauge group.

It is indeed much easier to obtain adjoint quarks than fundamental quarks from AdS/CFT derivations. For example, with the manifold $AdS_5 \times S^5$ the GFT is an $\mathcal{N} = 4$ supersymmetric $SU(N)$ gauge field theory in which the fermions are gauginos in the adjoint representation. This is sometimes called $\mathcal{N} = 4$ QCD and has considerable similarity with QCD. The two principal differences are the presence of supersymmetry and the fact that the quarks are in adjoint representations rather than in fundamental representations.

The reconciliation of these two differences with QCD can be arranged simultaneously by the following asymmetric orbifold procedure.

To break supersymmetry from $\mathcal{N} = 4$ to $\mathcal{N} = 0$ we replace the manifold $AdS_5 \times S^5$ by the orbifold $AdS_5 \times S^5/\Gamma$ where Γ is a finite subgroup of the $SU(4)$ isometry of S^5 and such that $\Gamma \not\subset SU(3) \subset SU(4)$ for any choice of the $SU(3)$ subgroup. This ensures that $\mathcal{N} = 0$ or no supersymmetry in the gauge field theory. There is no advantage in using non-abelian Γ so we choose abelian $\Gamma = Z_p$ which leads to a semi-simple gauge group $SU(N)^p$.

QCD has only non-chiral quarks so we need to choose an embedding with $\mathbf{4} \equiv \mathbf{4}^*$ of $SU(4)$. Defining $\alpha^p = 1$ or $\alpha = \exp(2\pi i/p)$ the embedding is defined by $\mathbf{4} = (\alpha^{A_1}, \alpha^{A_2}, \alpha^{A_3}, \alpha^{A_4})$ and for $\mathcal{N} = 0$ one requires all A_μ ($\mu = 1, 2, 3, 4$) to be non-zero and that the sum $\sum_{\mu=1}^{\mu=4} A_\mu = 0 \pmod{p}$. The fermions which survive the orbifolding are those which are invariant under a product of a Z_p transformation and a $SU(N)^p$ gauge transformation. These fermions are in the bifundamental representations of $SU(N)^p$:

$$\sum_{i=1}^{i=p} \sum_{\mu=1}^{\mu=4} (N_i, \bar{N}_{i+A_\mu}). \quad (1)$$

The complex scalars which survive are also those which are invariant under a product of a Z_p trans-

formation and a $SU(N)^p$ gauge transformation but now follows from the real $\mathbf{6} \equiv (v_i, v_i^*)$ of $SU(4)$ with $i = (1, 2, 3)$ and $v_i = (\alpha^{a_1}, \alpha^{a_2}, \alpha^{a_3})$. Here $a_1 = (A_2 + A_3)$, $a_2 = (A_3 + A_1)$ and $a_3 = (A_1 + A_2)$ although we note that the subscript orderings of A_μ, a_i are arbitrary and we can therefore choose $A_1 \leq A_2 \leq A_3 \leq A_4$ as well as $a_1 \leq a_2 \leq a_3$. The complex scalars now transform under $SU(N)^p$ as

$$\sum_{j=1}^{j=p} \sum_{i=1}^{i=3} (N_j, \bar{N}_{j \pm a_i}). \quad (2)$$

Both Eqs. (1) and (2) can be conveniently displayed in quiver or moose diagrams as we shall illustrate for $p = 3$ in Fig. 1.

For simplicity we choose the lowest possible value of p ($p = 3$) which works.

Note that for $p = 2$ the only choice for A_μ is $A_\mu = (1, 1, 1, 1)$ and consequently $a_i = (0, 0, 0)$ which means the quiver diagram for scalars, from Eq. (2), is disconnected into two separate $SU(N)$ s and no progress toward QCD is possible. Choosing $p = 3$ is, however, sufficient. In this case the unique choice to attain $\mathcal{N} = 0$ is $A_\mu = (1, 1, 2, 2)$ and therefore $a_i = (0, 0, 1)$. The scalar quiver is shown in Fig. 1(a). Each $SU(N)$ has two adjoints and two $(N + N^*)$ fundamentals of scalars.

If we now spontaneously break the $SU(N)^p$ symmetry in a way which respects the Z_p symmetries of the quiver diagram, and identify the QCD gauge group

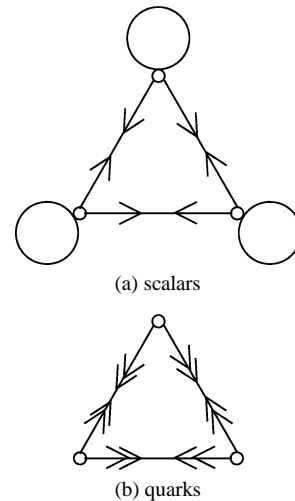


Fig. 1. Quiver diagrams.

as the diagonal subgroup of the three $SU(N)$ groups then the quarks will again be in adjoints just as in the $\mathcal{N} = 4$ case but without the supersymmetry.

Instead we choose an asymmetrical symmetry breaking where two of the $SU(N)$ s, say the two lower nodes in Fig. 1(a) are completely broken which is straightforward using VEVs of the available scalars in an asymmetric potential.

The third node, say the upper vertex of the triangular quiver in Fig. 1(a), is identified with the gauge group of QCD. There are two color adjoints and two fundamentals of scalars which can be made massive.

Finally, the chiral fermions are given by Eq. (1) and are depicted in the quiver diagram of Fig. 1(b). With respect to the unbroken QCD $SU(N)$ the fermions are non-chiral and occur in four flavors of $(N + N^*)$. No adjoint quarks appear.

Four flavors occur because of the **4** of the $SU(4)$ isotropy of S^5 . Increasing to $p \geq 4$ does not increase the number of flavors and has no other advantage so the $p = 3$ case of Fig. 1 is the simplest AdS/CFT model for nonsupersymmetric QCD with quarks in fundamental representations.

The dynamical studies in [9,10] presumably carry over to the present case since they depend only on the AdS geometry. Using the present AdS/CFT version of string QCD will, however, help arrange the correct color group theory factors to appear in the calculations.

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